Pg 357 11, 13, 15, 16, 17, 18, 19, 24

11

$$\frac{dy}{dt} = 1.5y, \quad y(0) = 100$$

$$\frac{1}{y}\,dy = 1.5\,dt$$

$$\ln y = 1.5t + C$$

$$y = e^{C} e^{1.5t}$$

$$y = Ae^{1.5t}$$

$$100 = Ae^{1.5(0)} = A$$

$$\underline{y = 100e^{1.5t}}$$

13

$$\frac{dy}{dt} = ky, \quad y(0) = 50, \quad y(5) = 100$$

$$\frac{1}{y}\,dy = k\,dt$$

$$\ln y = kt + C$$

$$y = Ae^{kt}$$

$$50 = Ae^{k(0)} = 50$$

$$y = 50e^{kt}$$

$$100 = 50e^{5k}$$

$$e^{5k} = 2$$

$$5k = \ln 2$$

$$k = \frac{\ln 2}{5}$$

$$\underline{y = 50e^{\frac{\ln 2}{5}t}}$$

15

a) double

$$2000 = 1000e^{.086t}$$

$$e^{.086t} = 2$$

$$.086t = \ln 2$$

$$t = \frac{\ln 2}{.086} \approx \underline{8.06 \text{ yrs}}$$

b) 30 yr

$$A = 1000e^{.086(30)}$$

$$\approx \underline{\$13{,}197.14}$$

16

a) rate

$$4000 = 2000e^{r \cdot 15}$$

$$e^{15r} = 2$$

$$15r = \ln 2$$

$$r = \frac{\ln 2}{15} \approx \underline{4.62\%}$$

b) 30 yr

$$A = 2000e^{.0462(30)}$$

$\approx \underline{\$7997.65}$ (rounded)

17

a) Initial

$$2898.44 = A_0 e^{.0525(30)}$$

$$A_0 = \frac{2898.44}{e^{.0525(30)}}$$

$$\approx \underline{\$600^{00}}$$

b) double

$$1200 = 600e^{.0525t}$$

$$t = \frac{\ln 2}{.0525}$$

$$\approx \underline{13.20 \text{ yrs}}$$

18 a) rate

$10{,}405.37 = 1200 e^{r(30)}$

$e^{30r} = \frac{10{,}405.37}{1200}$

$r = \frac{\ln\left(\frac{10{,}405.37}{1200}\right)}{30}$

$\approx \underline{7.2\%}$

b) Double

$2400 = 1200 e^{.072t}$

$t = \frac{\ln 2}{.072}$

$\approx \underline{9.63 \text{ yrs}}$

19

$A_0 = 2000, \quad r = 4.75\%$

a) $4000 = 2000\left(1 + \frac{.0475}{1}\right)^{t(1)}$

$2 = (1 + .0475)^t$

$t = \frac{\ln 2}{\ln(1 + .0475)} \approx \underline{14.94 \text{ yr}}$

b) $4000 = 2000\left(1 + \frac{.0475}{12}\right)^{12t}$

$t \approx \underline{14.62 \text{ yrs}}$

c) $4000 = 2000\left(1 + \frac{.0475}{4}\right)^{4t}$

$t \approx \underline{14.68 \text{ yrs}}$

d) $4000 = 2000 e^{.0475t}$

$t \approx \underline{14.59 \text{ yrs}}$

24

$10{,}000 = A_0 e^{3k}, \quad 40{,}000 = A_0 e^{5k}$

$A_0 = \frac{10{,}000}{e^{3k}} \qquad A_0 = \frac{40000}{e^{5k}}$

$\frac{40{,}000}{10{,}000} = \frac{e^{5k}}{e^{3k}}$

$4 = e^{2k}$

$\ln 4 = 2k$

$\underline{k = \ln 2}$

$10{,}000 = A_0 e^{3\ln 2}$

$A_0 = \frac{10{,}000}{e^{3\ln 2}} \approx \underline{1250}$

25

$.90 y_0 = y_0 e^{-0.18t}$

$.9 = e^{-.18t}$

$t = \frac{\ln .9}{-.18}$

$\approx \underline{.585} \text{ days}$

Pg 357 # 24, 29, 32, 41

(24) $y = y_0 e^{kt}$

$10{,}000 = y_0 e^{3k}$ $\quad 40{,}000 = y_0 e^{5k}$

$$\frac{40{,}000}{10{,}000} = \frac{y_0 e^{5k}}{y_0 e^{3k}}$$

$$4 = e^{5k-3k}$$

$$4 = e^{2k}$$

$$\ln 4 = 2k$$

$$k = \frac{\ln 4}{2} = \ln 2$$

$$10{,}000 = y_0 e^{3 \cdot \ln 2}$$

$$y_0 = \frac{10{,}000}{e^{3 \ln 2}} = \frac{10{,}000}{e^{\ln 8}}$$

$$= \frac{10{,}000}{8}$$

$$\boxed{y_0 = 1250}$$

(29) $y = y_0 e^{-kt}$

$$y_0 e^{-k(3/k)} = y_0 e^{-3}$$

$$= .0498\, y_0$$

This is less than 5% of original, which means over 95% has decayed

(32) $T - T_s = (T_0 - T_s) e^{-kt}$

$t = 0$, $T_0 - T_s = 60$

$$70 = 60 e^{-k(-20)}$$

$$k = \frac{\ln \frac{7}{6}}{20}$$

a) $T - T_s = (T_0 - T_s) e^{-kt}$

$$T - T_s = 60 e^{-\left(\frac{\ln \frac{7}{6}}{20}\right) 15}$$

$= 53.45°$ above room temp

b) $T - T_s = 60 e^{-\left(\frac{\ln \frac{7}{6}}{20}\right) 120}$

$= 23.79°C$ above room temp

c) $10 = 60 e^{-\left(\frac{\ln 7/6}{20}\right) t}$

$t = 232.47$ min ≈ 3.9 hrs

41

a) $\frac{dp}{dh} = k p$

$\frac{1}{p} dp = k \, dh$

$\ln|p| = kh + c$

$p = Ce^{kh}$ $\quad h=0,$ $\quad p = 1013$

$P_0 = Ce^{k(0)}$

$P_0 = C = 1013$

$p = 1013 e^{kh}$

at $h = 20$ $\quad p = 90$

$90 = 1013 e^{20k}$

$k = -.121$

$p = 1013 e^{-.121 h}$

b) $p(50) = 1013 e^{-.121(50)}$

$= 2.323$ millibars

c) $900 = 1013 e^{-.121 h}$

$h = .977$ km